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Implications of Super-Kamiokande atmospheric low-energy data for solar neutrino oscillations

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Abstract

It is shown that the high- Δm^2 part of the large mixing angle MSW solution of the solar neutrino problem is disfavored by the Super-Kamiokande data for low-energy upward-going events. A quantitative bound is obtained in the three-neutrino scheme with a negligibly small element U_{e3} of the neutrino mixing matrix, as indicated by the result of the CHOOZ long-baseline $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillation experiment.

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I. INTRODUCTION

The recent results of the Super-Kamiokande atmospheric neutrino experiment [1] provided a model-independent evidence in favor of neutrino oscillations that has been searched for since the discovery of the theory of neutrino oscillations [2]. The solar neutrino problem [3] and the atmospheric neutrino anomaly can be explained by neutrino oscillations in the scheme with mixing of the three flavor neutrinos ν_e , ν_μ and ν_τ with three massive neutrinos ν_1 , ν_2 and ν_3 . In this case the oscillations of solar and atmospheric neutrinos are due to the mass-squared differences $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$, respectively, where m_k is the mass of the massive neutrino ν_k ($k = 1, 2, 3$), and

$$\Delta m_{21}^2 \ll \Delta m_{31}^2. \quad (1.1)$$

The atmospheric neutrino anomaly has been observed in the Kamiokande [4], IMB [5], Soudan [6], Super-Kamiokande [1] and MACRO [7] experiments. The fit of the high-statistics Super-Kamiokande atmospheric neutrino data with two-neutrino $\nu_\mu \rightarrow \nu_\tau$ oscillations yielded an allowed region in the $\sin^2 2\vartheta_{\text{atm}} - \Delta m_{31}^2$ plane delimited by [1]

$$\sin^2 2\vartheta_{\text{atm}} \gtrsim 0.7, \quad 3 \times 10^{-4} \text{ eV}^2 \lesssim \Delta m_{31}^2 \lesssim 9 \times 10^{-3} \text{ eV}^2, \quad (1.2)$$

at 99% CL. Furthermore, the results of the Kamiokande [4], IMB [5] and Soudan [6] experiments, together with the preliminary new data of the Super-Kamiokande experiment [8], indicate that Δm_{31}^2 is larger than a few times 10^{-3} eV^2 . Hence, in the following we will consider

$$2 \times 10^{-3} \text{ eV}^2 \lesssim \Delta m_{31}^2 \lesssim 9 \times 10^{-3} \text{ eV}^2. \quad (1.3)$$

The rates of the solar neutrino experiments (Homestake [9], Kamiokande [10], GALLEX [11], SAGE [12] and Super-Kamiokande [13,14]) can be explained by $\nu_e \rightarrow \nu_\mu$ and/or $\nu_e \rightarrow \nu_\tau$ oscillations in vacuum or MSW [15] resonant transitions in the interior of the sun with a small or a large mixing angle ϑ_{sun} . Here we are interested in the large mixing angle MSW solution of the solar neutrino problem whose allowed region in the two-neutrino mixing parameter space $\sin^2 2\vartheta_{\text{sun}} - \Delta m_{21}^2$ is delimited by [16]

$$\sin^2 2\vartheta_{\text{sun}} \gtrsim 0.5, \quad 6 \times 10^{-6} \text{ eV}^2 \lesssim \Delta m_{21}^2 \lesssim 3 \times 10^{-4} \text{ eV}^2, \quad (1.4)$$

at 99% CL. These large values of the mass-squared difference Δm_{21}^2 and mixing angle $\sin^2 2\vartheta_{\text{sun}}$ are large enough to have an effect on upward-going low-energy atmospheric neutrinos. Indeed, these neutrinos have energy $100 \text{ MeV} \lesssim E \lesssim 1 \text{ GeV}$ and propagate for a distance $10^3 \text{ km} \lesssim L \lesssim 10^4 \text{ km}$. This means that the corresponding energy-dependent phase for neutrino oscillations is

$$\frac{\Delta m_{21}^2 L}{2E} \sim 1 \quad (1.5)$$

and hence must be taken into account in the analysis of atmospheric neutrino data [17].

If the disappearance of atmospheric ν_μ 's observed by the Super-Kamiokande and other atmospheric neutrino experiments is due to $\nu_\mu \rightarrow \nu_\tau$ oscillations driven by the mass-squared

difference Δm_{31}^2 , the high- Δm_{21}^2 part of the large mixing angle MSW solution of the solar neutrino problem is disfavored by the Super-Kamiokande data relative to low-energy upward-going e -like and μ -like events. Indeed, Eq.(1.5) implies that low-energy upward-going neutrinos undergo not only $\nu_\mu \rightarrow \nu_\tau$ transitions driven by Δm_{31}^2 but also $\nu_\mu \rightleftharpoons \nu_e$, $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\tau$ transitions driven by Δm_{21}^2 . The occurrence of significant $\nu_\mu \rightleftharpoons \nu_e$ and $\nu_\mu \rightarrow \nu_\tau$ transitions due to Δm_{21}^2 is excluded by the absence of an additional deficit of low-energy upward-going μ -like events with respect to the main deficit due to $\nu_\mu \rightarrow \nu_\tau$ oscillations driven by Δm_{31}^2 (see Fig.3 of Ref. [1]). The occurrence of $\nu_e \rightarrow \nu_\tau$ transitions is excluded by the absence of a deficit of low-energy upward-going e -like events (see Fig.3 of Ref. [1]). Let us notice that the introduction in the neutrino mixing scheme of additional sterile neutrinos (see [18] and references therein) required to explain also the indications in favor of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ transitions found in the LSND experiment [19] makes things even worse, because active neutrinos can have additional transitions into sterile states, leading to additional unobserved deficits of both e -like and μ -like events.

In this paper we will present a quantitative proof of the fact that the high- Δm^2 part of the large mixing angle MSW solution of the solar neutrino problem is disfavored by the Super-Kamiokande atmospheric neutrino data assuming the scheme of neutrino mixing indicated by the results of solar and atmospheric neutrino experiments together with the result of the long-baseline experiment reactor neutrino oscillation experiment CHOOZ [20]. In this scheme the element U_{e3} of the neutrino mixing matrix is negligibly small [21,22] and the transition probabilities of atmospheric neutrinos can be calculated analytically. We will show that in the scheme under consideration the high- Δm^2 part of the large mixing angle MSW solution of the solar neutrino problem is disfavored because it implies a depletion of atmospheric upward-going low-energy e -like events, which is in contradiction with the observations of the Super-Kamiokande experiment.

Our starting point is the fact that under the hypothesis of three-flavour neutrino oscillations, the number of e -like events, N_e^{DATA} , and the number of μ -like events, N_μ^{DATA} , measured in the Super-Kamiokande experiment are given by

$$N_e^{\text{DATA}} = N_e^{\text{MC}} + (N_\mu^{\text{MC}} - N_e^{\text{MC}}) P_{e\mu} - N_e^{\text{MC}} P_{e\tau}, \quad (1.6)$$

$$N_\mu^{\text{DATA}} = N_\mu^{\text{MC}} - (N_\mu^{\text{MC}} - N_e^{\text{MC}}) P_{e\mu} - N_\mu^{\text{MC}} P_{\mu\tau}, \quad (1.7)$$

where N_e^{MC} and N_μ^{MC} are the number of events calculated with the Monte Carlo method, without neutrino oscillations, and $P_{\alpha\beta}$ is the probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions properly averaged over the neutrino energy spectrum, the neutrino propagation distance, and the relative amounts of neutrinos and antineutrinos. Here we assume that, at least after these averagings, the probabilities of $\nu_\alpha \rightarrow \nu_\beta$ and $\nu_\beta \rightarrow \nu_\alpha$ transitions are equal.

II. THE NEUTRINO MIXING SCHEME

In the scheme with three-neutrino mixing where Δm_{21}^2 is responsible for solar neutrino oscillations and $\Delta m_{31}^2 \gg \Delta m_{21}^2$ is responsible for atmospheric neutrino oscillations, the negative result of the CHOOZ experiment in the search for long-baseline $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillations implies that

$$\sin^2 2\vartheta_{\text{CHOOZ}} \leq 0.18 \quad \text{for} \quad \Delta m_{31}^2 \gtrsim 2 \times 10^{-3} \text{ eV}^2. \quad (2.1)$$

Hence, this limit applies to the range (1.3) of Δm_{31}^2 under consideration.

In the three-neutrino scheme with $\Delta m_{31}^2 \gg \Delta m_{21}^2$ we have

$$\sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \quad (2.2)$$

(see [23,24]), where U is the 3×3 neutrino mixing matrix. The upper bound (2.1) on $\sin^2 2\vartheta_{\text{CHOOZ}}$ implies that

$$|U_{e3}|^2 \leq 5 \times 10^{-2} \quad \text{or} \quad |U_{e3}|^2 \geq 0.95. \quad (2.3)$$

A large value of $|U_{e3}|^2$ fails to explain the solar neutrino problem with neutrino oscillations because in the scheme under consideration the averaged survival probability of solar electron neutrinos is given by [25]

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) = (1 - |U_{e3}|^2)^2 P_{\nu_e \rightarrow \nu_e}^{(1,2)}(E) + |U_{e3}|^4, \quad (2.4)$$

where E is the neutrino energy and $P_{\nu_e \rightarrow \nu_e}^{(1,2)}(E)$ is the survival probability of solar ν_e 's due to the mixing of ν_e with ν_1 and ν_2 . The expression (2.4) implies that $P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \geq |U_{e3}|^4$ and for $|U_{e3}|^2 \geq 0.95$ we have $P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \geq 0.90$. With such a high and practically constant value of $P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E)$ it is not possible to explain the suppression of the solar ν_e flux measured by all experiments (Homestake [9], Kamiokande [10], GALLEX [11], SAGE [12] and Super-Kamiokande [13,14]) with respect to that predicted by the Standard Solar Model (see [26] and references therein). Therefore, the results of the CHOOZ experiment together with the results of solar and atmospheric neutrino experiments, imply that $|U_{e3}|^2$ is small:

$$|U_{e3}|^2 \leq 5 \times 10^{-2}. \quad (2.5)$$

Such a small value of $|U_{e3}|^2$ implies that the oscillations of solar and atmospheric neutrinos are decoupled [22] and the two-generation analyses of the solar neutrino data yield correct information on the values of Δm_{21}^2 and $\sin^2 2\vartheta_{\text{sun}} \simeq |U_{e2}|^2$.

Furthermore, it has been shown in Ref. [22] that if not only $|U_{e3}|^2$ is small, but also $|U_{e3}| \ll 1$, the two-generation analyses of the atmospheric neutrino data with $\nu_\mu \rightarrow \nu_\tau$ oscillations yield correct information on the values of Δm_{31}^2 and $\sin^2 2\vartheta_{\text{atm}} = 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)$. Hence, in the following we will assume that $|U_{e3}| \ll 1$ [21,22]. In this case the neutrino mixing matrix can be written as

$$U \simeq \begin{pmatrix} \cos \vartheta_{\text{sun}} & \sin \vartheta_{\text{sun}} & 0 \\ -\sin \vartheta_{\text{sun}} \cos \vartheta_{\text{atm}} & \cos \vartheta_{\text{sun}} \cos \vartheta_{\text{atm}} & \sin \vartheta_{\text{atm}} \\ \sin \vartheta_{\text{sun}} \sin \vartheta_{\text{atm}} & -\cos \vartheta_{\text{sun}} \sin \vartheta_{\text{atm}} & \cos \vartheta_{\text{atm}} \end{pmatrix}. \quad (2.6)$$

A particular case of this mixing matrix is obtained for $\vartheta_{\text{sun}} = \vartheta_{\text{atm}} = \pi/4$ and corresponds to the bi-maximal mixing that has been assumed recently by several authors [27]. Notice, however, that bi-maximal mixing is not compatible with the large mixing angle MSW solution of the solar neutrino problem [28].

The assumption $|U_{e3}| \ll 1$ implies that CP and T violation in the lepton sector is negligibly small. Indeed, the CP-violating phase that is present in the general expression of the mixing matrix (see [29]) can be eliminated if one of the elements of the mixing matrix is zero (this can also be seen by noticing that in this case the Jarlskog rephasing-invariant parameter [30] is equal to zero). Hence, the probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions in vacuum is the same as that of $\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta$ transitions, but the probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions in matter is different from that of $\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta$ transitions because neutrino and antineutrinos have different interactions with the medium, which induce different effective potentials. On the other hand, the probabilities of $\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta$ and $\overset{(-)}{\bar{\nu}}_\beta \rightarrow \overset{(-)}{\bar{\nu}}_\alpha$ transitions in vacuum and in matter are the same because T is conserved in the lepton sector and the matter distribution along a neutrino path crossing the Earth is (approximately) symmetrical. For simplicity, in the following we will neglect the matter effect for the oscillations of atmospheric neutrinos and we will not make a distinction between neutrinos and antineutrinos. The contribution of the matter effect will be discussed elsewhere [31].

The probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions in vacuum is given by

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| U_{\alpha 1} U_{\beta 1} + U_{\alpha 2} U_{\beta 2} \exp \left(-i \frac{\Delta m_{21}^2 L}{2E} \right) + U_{\alpha 3} U_{\beta 3} \exp \left(-i \frac{\Delta m_{31}^2 L}{2E} \right) \right|^2. \quad (2.7)$$

Let us consider upward-going low-energy neutrinos with

$$0.1 \text{ GeV} \lesssim E \lesssim 1 \text{ GeV}, \quad 10^3 \lesssim L \lesssim 10^4 \text{ km}. \quad (2.8)$$

Taking into account the value (1.3) for Δm_{31}^2 , we have

$$\frac{\Delta m_{31}^2 L}{E} \gtrsim 10. \quad (2.9)$$

These large values of $\Delta m_{31}^2 L/2E$ imply that the oscillations generated by the mass-squared difference Δm_{31}^2 are very rapid so that $\sin^2 \Delta m_{31}^2 L/2E$ can be set to 1/2. Therefore, the measured probability is given by the average of the expression (2.7) over the fast oscillations due to Δm_{31}^2 :

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| U_{\alpha 1} U_{\beta 1} + U_{\alpha 2} U_{\beta 2} \exp \left(-i \frac{\Delta m_{21}^2 L}{2E} \right) \right|^2 + U_{\alpha 3}^2 U_{\beta 3}^2, \quad (2.10)$$

which can be written as

$$P_{\nu_\alpha \rightarrow \nu_\beta} = (1 - U_{\alpha 3}^2) (1 - U_{\beta 3}^2) P_{\nu_\alpha \rightarrow \nu_\beta}^{(1,2)} + U_{\alpha 3}^2 U_{\beta 3}^2, \quad (2.11)$$

where

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}^{(1,2)} &= \left(\frac{U_{\alpha 1}^2}{1 - U_{\alpha 3}^2} \right) \left(\frac{U_{\beta 1}^2}{1 - U_{\beta 3}^2} \right) + \left(\frac{U_{\alpha 2}^2}{1 - U_{\alpha 3}^2} \right) \left(\frac{U_{\beta 2}^2}{1 - U_{\beta 3}^2} \right) \\ &\quad + 2 \left(\frac{U_{\alpha 1} U_{\alpha 2}}{1 - U_{\alpha 3}^2} \right) \left(\frac{U_{\beta 1} U_{\beta 2}}{1 - U_{\beta 3}^2} \right) \cos \frac{\Delta m_{21}^2 L}{2E} \end{aligned} \quad (2.12)$$

is the probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions due to the mixing of ν_α and ν_β with ν_1 and ν_2 . From Eq.(2.6) we have

$$\frac{U_{e1}^2}{1 - U_{e3}^2} = \cos^2 \vartheta_{\text{sun}}, \quad \frac{U_{e2}^2}{1 - U_{e3}^2} = \sin^2 \vartheta_{\text{sun}}, \quad \frac{U_{e1} U_{e2}}{1 - U_{e3}^2} = \frac{1}{2} \sin 2\vartheta_{\text{sun}} \quad (2.13)$$

and

$$\frac{U_{\alpha 1}^2}{1 - U_{\alpha 3}^2} = \sin^2 \vartheta_{\text{sun}}, \quad \frac{U_{\alpha 2}^2}{1 - U_{\alpha 3}^2} = \cos^2 \vartheta_{\text{sun}}, \quad \frac{U_{\alpha 1} U_{\alpha 2}}{1 - U_{\alpha 3}^2} = -\frac{1}{2} \sin 2\vartheta_{\text{sun}} \quad (2.14)$$

for $\alpha = \mu, \tau$. Hence, the probabilities $P_{\nu_\alpha \rightarrow \nu_\beta}^{(1,2)}$ have a two-generation form and depend only on the two parameters relevant for solar neutrino oscillations, Δm_{21}^2 and ϑ_{sun} :

$$P_{\nu_\alpha \rightarrow \nu_\alpha}^{(1,2)} = P_{\nu_\mu \leftrightarrow \nu_\tau}^{(1,2)} = 1 - P_{21}, \quad P_{\nu_e \leftrightarrow \nu_\beta}^{(1,2)} = P_{21} \quad (\alpha = e, \mu, \tau; \beta = \mu, \tau), \quad (2.15)$$

with

$$P_{21} = \frac{1}{2} \sin^2 2\vartheta_{\text{sun}} \left(1 - \cos \frac{\Delta m_{21}^2 L}{2E} \right). \quad (2.16)$$

The complete expressions for the transition probabilities are¹:

$$P_{\nu_e \rightarrow \nu_e} = 1 - P_{21}, \quad (2.17)$$

$$P_{\nu_\mu \rightarrow \nu_\mu} = 1 - \frac{1}{2} \sin^2 2\vartheta_{\text{atm}} - \cos^4 \vartheta_{\text{atm}} P_{21}, \quad (2.18)$$

$$P_{\nu_\tau \rightarrow \nu_\tau} = 1 - \frac{1}{2} \sin^2 2\vartheta_{\text{atm}} - \sin^4 \vartheta_{\text{atm}} P_{21}, \quad (2.19)$$

$$P_{\nu_e \leftrightarrow \nu_\mu} = \cos^2 \vartheta_{\text{atm}} P_{21}, \quad (2.20)$$

$$P_{\nu_e \leftrightarrow \nu_\tau} = \sin^2 \vartheta_{\text{atm}} P_{21}, \quad (2.21)$$

$$P_{\nu_\mu \leftrightarrow \nu_\tau} = \frac{1}{4} \sin^2 2\vartheta_{\text{atm}} (2 - P_{21}). \quad (2.22)$$

It is important to notice that the probability of $\nu_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow \nu_\tau$ transitions are approximately equal if $\sin^2 2\vartheta_{\text{atm}} \simeq 1$, as indicated by the $\nu_\mu \rightarrow \nu_\tau$ fit of all the atmospheric contained events and upward-going muons measured in the Super-Kamiokande experiment [8]. In this case, if $P_{21} \neq 0$ the disappearance of electron neutrinos due to $\nu_e \rightarrow \nu_\tau$ transitions cannot be compensated by $\nu_e \leftrightarrow \nu_\mu$ oscillations. This can be seen by examining the ratio

$$\frac{N_e^{\text{DATA}}}{N_e^{\text{MC}}} = 1 - (1 - R_{\mu/e}^{\text{MC}}) P_{e\mu} - P_{e\tau}, \quad (2.23)$$

where $R_{\mu/e}^{\text{MC}} \equiv N_\mu^{\text{MC}}/N_e^{\text{MC}}$. For $\sin^2 2\vartheta_{\text{atm}} \simeq 1$ we have $P_{e\tau} \simeq P_{e\mu} \simeq \frac{1}{2} \langle P_{21} \rangle$, where $\langle P_{21} \rangle$ indicates the average of P_{21} over the energy spectrum and propagation distance of atmospheric neutrinos. It follows that

¹We would like to thank A.Yu. Smirnov for noticing a mistake in the expression of $P_{\nu_e \leftrightarrow \nu_\tau}$ presented in the first version of this paper.

$$\frac{N_e^{\text{DATA}}}{N_e^{\text{MC}}} \simeq 1 - \frac{1}{2} (2 - R_{\mu/e}^{\text{MC}}) \langle P_{21} \rangle. \quad (2.24)$$

Since $R_{\mu/e}^{\text{MC}} < 2$ for low-energy events in the Super-Kamiokande experiment, one can see that N_e^{DATA} can only *decrease* with respect to N_e^{MC} if $\sin^2 2\vartheta_{\text{atm}} \simeq 1$.

The expression of $N_e^{\text{DATA}}/N_e^{\text{MC}}$ valid for any value of $\sin^2 2\vartheta_{\text{atm}}$ is

$$\frac{N_e^{\text{DATA}}}{N_e^{\text{MC}}} = 1 - (1 - R_{\mu/e}^{\text{MC}} \cos^2 \vartheta_{\text{atm}}) \langle P_{21} \rangle. \quad (2.25)$$

Therefore, one can see that a non-zero value of $\langle P_{21} \rangle$ implies that N_e^{DATA} should be smaller than N_e^{MC} if $\sin^2 \vartheta_{\text{atm}} > (R_{\mu/e}^{\text{MC}})^{-1}$.

III. CONSTRAINT ON THE LARGE MIXING ANGLE MSW SOLUTION OF THE SOLAR NEUTRINO PROBLEM

An experimental lower limit

$$\frac{N_e^{\text{DATA}}}{N_e^{\text{MC}}} \geq R_e^{\text{min}} \quad (3.1)$$

relative to low-energy upward-going e -like atmospheric neutrino events allows to constraint the value of $\langle P_{21} \rangle$. Indeed, using Eq.(2.25) we have

$$\langle P_{21} \rangle \leq \frac{1 - R_e^{\text{min}}}{1 - R_{\mu/e}^{\text{MC}} \cos^2 \vartheta_{\text{atm}}}. \quad (3.2)$$

This implies that only the region in the $\sin^2 2\vartheta_{\text{sun}} - \Delta m_{21}^2$ such that

$$\sin^2 2\vartheta_{\text{sun}} \leq \frac{2 (1 - R_e^{\text{min}})}{\left(1 - R_{\mu/e}^{\text{MC}} \cos^2 \vartheta_{\text{atm}}\right) \left(1 - \left\langle \cos \frac{\Delta m_{21}^2 L}{2E} \right\rangle\right)} \quad (3.3)$$

is allowed. The brackets around the cosine indicate an appropriate average over energy and distance.

The Super-Kamiokande data relative to upward-going ($\cos \theta < -0.2$, where θ is the zenith angle) e -like and μ -like events with momentum $p < 0.4\text{GeV}$ are [1]:

$$N_e^{\text{DATA}} = 272 \pm 23, \quad (3.4)$$

$$N_e^{\text{MC}} = 247 \pm 10, \quad (3.5)$$

$$N_{\mu}^{\text{DATA}} = 183 \pm 19, \quad (3.6)$$

$$N_{\mu}^{\text{MC}} = 313 \pm 10, \quad (3.7)$$

where the MC expected fluxes have been rescaled by a factor 1.158 with respect to the fluxes calculated in Ref. [32] (which predict $N_e^{\text{MC}} = 214 \pm 8$ and $N_{\mu}^{\text{MC}} = 270 \pm 9$), as required by the fit of all the Super-Kamiokande data with $\nu_{\mu} \rightarrow \nu_{\tau}$ oscillations [1].

The ratio $R_{\mu/e}^{\text{MC}}$ of expected μ -like and e -like events is given by

$$R_{\mu/e}^{\text{MC}} \equiv \frac{N_{\mu}^{\text{MC}}}{N_e^{\text{MC}}} = 1.27 \pm 0.06. \quad (3.8)$$

The error of the theoretical calculation is about 5% and will be neglected in the following approximate calculations. Since $(R_{\mu/e}^{\text{MC}})^{-1} = 0.79$, as discussed at the end of the previous Section, a value of $\langle P_{21} \rangle$ bigger than zero implies that N_e^{DATA} should be smaller than N_e^{MC} if $\sin^2 \vartheta_{\text{atm}} > 0.21$, which is practically guaranteed to be certain by the result of the fit of all Super-Kamiokande atmospheric neutrino data with $\nu_{\mu} \rightarrow \nu_{\tau}$ transitions [1,8].

From the data in Eqs.(3.4) and (3.5), the value of the ratio (3.1) is

$$\frac{N_e^{\text{DATA}}}{N_e^{\text{MC}}} = 1.10 \pm 0.10 \geq 0.97 \quad \text{at 90\% CL.} \quad (3.9)$$

Therefore, we consider

$$R_e^{\text{min}} = 0.97. \quad (3.10)$$

If we consider now the fit of all Super-Kamiokande data with neutrino oscillations, the value of $\sin^2 2\vartheta_{\text{atm}}$ is constrained to be bigger than about 0.90 at 90% CL [8]. Hence, we have

$$0.34 \lesssim \cos^2 \vartheta_{\text{atm}} \lesssim 0.66. \quad (3.11)$$

Inserting the values (3.8), (3.10) and (3.11) in the inequality (3.3) we obtain

$$\sin^2 2\vartheta_{\text{sun}} \leq \frac{0.37}{1 - \left\langle \cos \frac{\Delta m_{21}^2 L}{2E} \right\rangle}. \quad (3.12)$$

The corresponding exclusion curve in the $\sin^2 2\vartheta_{\text{sun}} - \Delta m_{21}^2$ plane obtained with an average of $\cos \frac{\Delta m_{21}^2 L}{2E}$ over a constant energy spectrum in the interval $100 \text{ MeV} \leq E \leq 1 \text{ GeV}$ and a constant angular distribution $-1 \leq \cos \theta \leq -0.2$, where θ is the zenith angle, is shown in Fig. 1 (solid curve, the region on the right is forbidden). The dotted line in Fig. 1 represents the exclusion curve obtained for $\sin^2 2\vartheta_{\text{atm}} = 1$ (the corresponding value of the numerator in Eq.(3.12) is 0.16). The light and dark shadowed areas in Fig. 1 represent the 99% CL allowed region of the large mixing angle MSW solution of the solar neutrino problem obtained in Ref. [16] from the fit of the total rates of all the solar neutrino experiments (Fig. 2). The region below the dashed line is excluded from the day-night asymmetry measured in the Super-Kamiokande experiment [13,14] (Fig. 10b of Ref. [16]) and the dark shadowed region is the 99% CL allowed region obtained in Ref. [16] (Fig. 10b) from the fit of the rates of all solar neutrino experiments plus the day-night asymmetry measured in the Super-Kamiokande experiment.

Let us emphasize that the limit (3.12) has been obtained under several approximations whose validity cannot rigorously be proved without more precise calculations [31]. Hence, the exclusion curve in Fig. 1 must be considered as an indication of disfavoring the high- Δm_{21}^2

part of the allowed region for the large mixing angle MSW solution of the solar neutrino problem. From Fig. 1 one can see that the solar plus atmospheric data of the Super-Kamiokande experiment, taking into account the non-observation of a day-night asymmetry in the solar data, disfavor practically all the allowed region of the large mixing angle MSW solution of the solar neutrino problem.

IV. CONCLUSIONS

We have considered the scheme with mixing of three massive neutrinos indicated by the results of atmospheric and solar neutrino experiments, together with the negative result of the CHOOZ reactor long-baseline experiment. In this scheme the element U_{e3} of the neutrino mixing matrix is negligibly small.

We have shown that if the scheme under consideration is realized in nature, the high- Δm^2 values of the large mixing angle MSW solution of the solar neutrino problem lead to a deficit of low-energy upward-going atmospheric e -like events with respect to the $\nu_\mu \rightarrow \nu_\tau$ fit of the experimental data. Since this deficit has not been observed in the Super-Kamiokande experiment, for on the contrary measured a slight excess of low-energy upward-going atmospheric e -like events, we conclude that the high- Δm^2 part of the large mixing angle MSW solution of the solar neutrino problem is disfavored by the atmospheric neutrino data of the Super-Kamiokande experiment. We have presented in Fig. 1 an approximate exclusion curve obtained from the Super-Kamiokande data which shows that the high- Δm^2 part of the large mixing angle MSW solution of the solar neutrino problem is disfavored. Taking into account the fact that the small- Δm^2 part of the large mixing angle MSW solution of the solar neutrino problem is excluded by the absence of a day-night asymmetry in the Super-Kamiokande solar neutrino data, we conclude that the large mixing angle MSW solution of the solar neutrino problem is disfavored by the solar and atmospheric neutrino data of the Super-Kamiokande experiment.

As discussed in Section I, we think that the large mixing angle MSW solution of the solar neutrino problem is disfavored by the data of the Super-Kamiokande experiment in any scheme of neutrino mixing. It would be very interesting to check this conclusion performing a combined fit of the atmospheric and solar neutrino data in the general framework of three-neutrino mixing.

It is worthwhile to emphasize that the large mixing angle MSW solution of the solar neutrino problem is also disfavored by the global fit of solar neutrino data, which includes the total rates measured in solar neutrino experiments and the energy spectrum and zenith angle distribution of recoil electrons measured in the Super-Kamiokande experiment (see Fig. 15(b) of Ref. [16]).

A more detailed analysis of the atmospheric neutrino data will be presented elsewhere [31].

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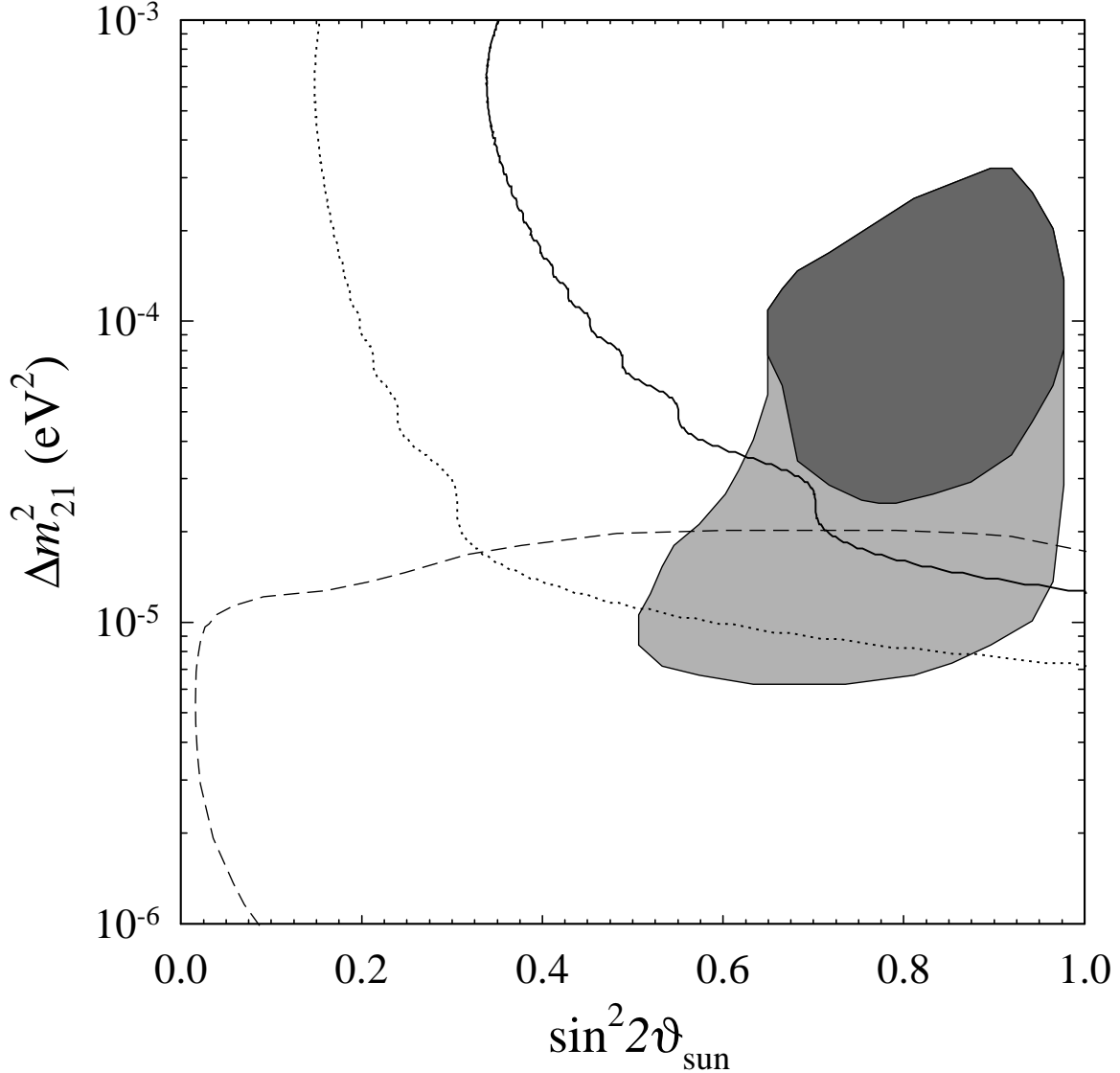


FIG. 1. Exclusion curve in the $\sin^2 2\vartheta_{\text{sun}} - \Delta m_{21}^2$ plane obtained from the Super-Kamiokande data relative to low-energy upward-going e -like events (solid line, the region on the right is forbidden). The dotted line represents the exclusion curve for $\sin^2 2\vartheta_{\text{atm}} = 1$. The light and dark shadowed areas represent the 99% CL allowed region of the large mixing angle MSW solution of the solar neutrino problem obtained from the fit of the total rates of all solar neutrino experiments (taken from Fig. 2 of Ref. [16]). The region below the dashed line is excluded from the day-night asymmetry measured in the Super-Kamiokande experiment [13,14] (Fig. 10b of Ref. [16]) and the dark shadowed region is the 99% CL allowed region obtained in Ref. [16] (Fig. 10b) from the fit of the rates of all solar neutrino experiments plus the day-night asymmetry measured in the Super-Kamiokande experiment.